Using the values obtained for radiative zone in eqn (26)

$$\langle N \rangle = \frac{c p_0^2 k^4}{32\pi^2 \varepsilon_0} \frac{\sin^2\theta}{r^2} \hat{e}_r \quad \text{--- (28)}$$

The total radiated power

$$W = \int_0^{2\pi}\int_0^{\pi} \frac{c p_0^2 k^4}{32\pi^2 \varepsilon_0} \frac{\sin^2\theta}{r^2} r^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{c p_0^2 k^4}{32\pi^2 \varepsilon_0} 2\pi \frac{4}{3} = \frac{c p_0^2 k^4}{12\pi \varepsilon_0} = \frac{c p_0^2}{12\pi\varepsilon_0}\left(\frac{2\pi}{\lambda}\right)^4 \quad \text{--- (29)}$$

Thus, the power radiated varies directly as the square of the amplitude of the electric dipole and inversely as the fourth power of the wavelength.

Radiation resistance?

When a current $I = I_0 e^{i\omega t}$ passes through a circuit containing a resistance R, the average rate of dissipation of energy is given by

$$\text{Rate of energy dissipated} = \frac{1}{2} R I_0^2 \quad \text{--- (30)}$$

eqn (29), the energy loss can be written as

$$\frac{c p_0^2}{12\pi\varepsilon_0}\left(\frac{2\pi}{\lambda}\right)^4 = \frac{p_0^2 \omega^4}{12\pi\varepsilon_0 c^3} = \frac{1}{12\pi\varepsilon_0} \frac{I_0^2 l^2 \omega^2}{c^3}$$

(from eqn 16)

$$= \frac{1}{12\pi\varepsilon_0} \frac{l^2 4\pi^2}{\lambda^2} \frac{1}{c} I_0^2$$

$$= \frac{1}{2} \frac{2\pi}{3} \sqrt{\frac{\mu_0}{\varepsilon_0}} \left(\frac{l}{\lambda}\right)^2 I_0^2 \quad \text{--- (31)}$$

Comparing eqn (31) with eqn (30) we find for the radiation resistance

$$R_r = \frac{2\pi}{3}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(\frac{l}{\lambda}\right)^2$$

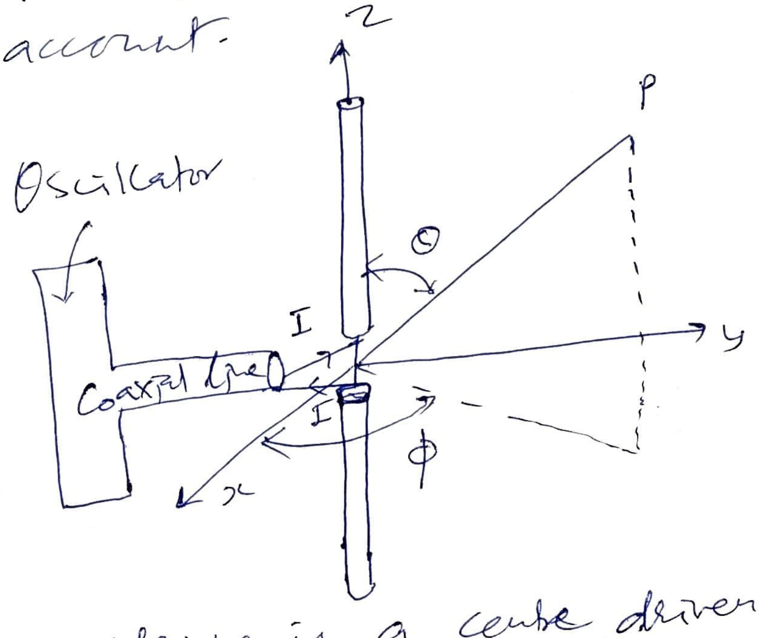
$$= 789\left(\frac{l}{\lambda}\right)^2 \text{ ohms} \quad \text{--- (32)}$$

The expression is valid provided $l \ll \lambda$.

<u>Linear Antenna</u> : We have derived eq^{n}s for systems with linear dimensions much smaller than the wavelength.

Antenna → used in radio or television transmission are generally not short compared to the wavelength of the radiation they transmit.

Current in the antenna is not constant and the variation of its amplitude must be taken into account.



A simple antenna is a centre driven linear antenna. The current is feed into the antenna (usually a very thin wire) via a coaxial cable trans-mission line. The antenna is assumed to be oriented along the z-axis and has a length 'l'. The antenna is excited across a small gap at the mid-point which coincides with the origin.

The current density is assumed to vary harmonically in time and space along the antenna.

Standing waves are set-up with the conditions that the current at each end is zero.
To a good approximation the current density can be written as

$$\vec{j}(r,t) = \hat{e}_z I_0 \exp(i\omega t) \sin\left(\frac{kl}{2} - k|z|\right)\delta x \delta y \quad \text{---(1)}$$

The delta function assure that the current flows in z-direction only. This expression satisfies this condition.

The current at the gap, i.e. the input signal is

$$I(t) = I_0 \exp(i\omega t) \sin\frac{kl}{2} \quad \text{--(2)}$$

The vector potential at a point P specified by the vector 'r' is

$$A(r,t) = \frac{\mu_0}{4\pi}\int \frac{j(r', t')}{|r - r'|} dr'$$

where $t' = t - \frac{|r - r'|}{c}$

$$= \frac{\mu_0}{4\pi} \hat{e}_z I_0 \int_{-l/2}^{l/2} \frac{e^{i\omega\left(t - \frac{|r - r'|}{c}\right)} \sin\left(\frac{kl}{2} - k|z|\right)}{|r - r'|} dr'$$

$$= \frac{\mu_0}{4\pi} \hat{e}_z I_0 \int_{-l/2}^{l/2} \frac{e^{i\omega\left(t - \frac{|r - z|}{c}\right)} \sin\left(\frac{kl}{2} - k|z|\right)}{|r - z|} dz \quad \text{--(3)}$$

If the point P is far away we can replace the denominator by r and in the numerator we may substitute

$$|r - z| = r - z\cos\theta$$

where θ is the angle the vector r makes with the z-axis.

Therefore

$$A(r,t) = \frac{\mu_0}{4\pi} \hat{e}_z \frac{I_0}{r} \exp(i\omega t)\, e^{-ikr} \times$$

$$\int_{-l/2}^{l/2} \sin\left(\frac{kl}{2} - k|z|\right) e^{ikz\cos\theta}\, dz$$

Integrating, we get

$$A(r,t) = \frac{\mu_0}{2\pi} \hat{e}_z I_0 \exp(i\omega t) \frac{e^{-ikr}}{kr} \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin^2\theta}\right] \quad \text{--- (4)}$$

One can easily calculate from this the field E and H. The average rate of power flow is given by

$$\langle N \rangle = \frac{1}{2} \mathrm{Re}\, (E \times H^*)$$

$$= \frac{\hat{e}_n}{2c\mu_0} |E_0|^2$$

$$= \frac{1}{2} \sqrt{\frac{\mu_0}{\varepsilon_0}}\, \hat{e}_n \frac{I_0^2}{4\pi^2} \frac{1}{r^2} \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta}\right]^2 \quad \text{--- (5)}$$

The average power radiated into unit solid angle

$$\left\langle \frac{dW}{d\Omega} \right\rangle = \frac{4\pi r^2 \langle N \rangle}{4\pi} \cdot \hat{e}_n = r^2 \langle N \rangle \cdot \hat{e}_n$$

$$= \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{I_0^2}{8\pi^2} \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta}\right]^2 \quad \text{--- (6)}$$

The angular distribution, therefore, depends upon the value of $kl/2$.